

Signal Processing

At the end of this Session, you will be able to:

- **Define the terms Amplitude, Frequency and Phase**
- **Define the terms 'phase lead' and 'phase lag'**
- **Define the terms 'zero-phase wavelet' and 'minimum-phase wavelet'**
- **Explain the difference between a continuous and a discrete signal**
- **Define the term 'sampling' and describe how it's governed by the sampling theorem.**
- **Define the terms 'spatial sampling' and 'temporal sampling'**
- **Define the term 'Nyquist frequency' and describe how it can be calculated for a given sampling interval.**
- **Define the term 'aliasing' and describe how you would calculate an aliased frequencies new value for a given sample interval**

At the end of this Session, you will be able to:

- Describe the relationship between Nyquist frequency and aliasing
- Describe at least 3 ways in which temporal aliasing can decrease the quality of seismic data
- Define the term 'Nyquist wavenumber' and describe how it can be calculated for a given spatial sampling interval.
- Define the term 'lateral (spatial) resolution'
- Define the term 'vertical (temporal) resolution'.
- Describe the Rayleigh criteria of resolution and show how it can be used to calculate vertical resolution.
- Define the term 'Fresnel zone' and calculate its radius for given velocity, frequency and travel time.

At the end of this Session, you will be able to:

- Define the terms 'time domain' and 'frequency domain'.
- Define the term 'Fourier series'.
- Define the terms 'fundamental frequency' and 'harmonic frequency'.
- Define the terms 'amplitude spectrum' and 'phase spectrum' and describe their components.
- Describe and compare the definitions of each of the following transforms: Fourier Transform, Discrete Fourier Transform and Fast Fourier Transform.
- Match time domain representations with their frequency domain equivalents

At the end of this Session, you will be able to:

- Define the term 'correlation' and describe the meaning of 'auto-correlation' and 'cross correlation'
- Describe how to correlate two wavelets
- Describe the equivalent of correlation in the frequency domain
- Describe how cross correlation and auto-correlation can be used in data processing
- Define the term 'convolution'
- Describe how to convolve two wavelets
- Describe the equivalent of convolution in the frequency domain
- Describe how convolution can be used in data processing

Signal Processing

- **Waves**
- **Data sampling**
- **Seismic resolution**
- **Time and Frequency domain representation**
- **Convolution and Correlation**

At the end of this Session, you will be able to:

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Waves

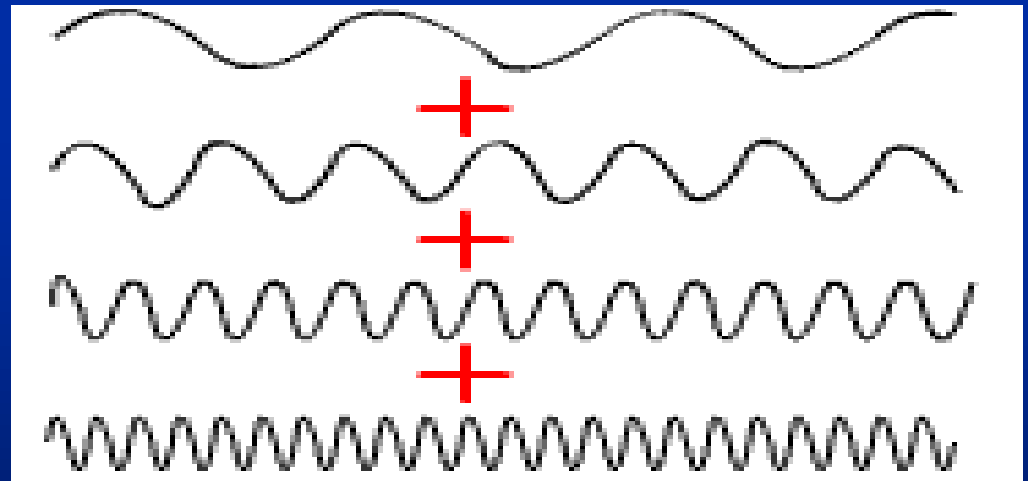
- Lets recap from our earlier Geophysics lecture. The different components of a wave.
- Waves

Waves - Amplitude

- Amplitude is the maximum departure of a wave from the average.
- The amplitudes down a seismic trace are dependent on the impedance contrast at each of the reflectors.
- The bigger the difference between the rocks on either side of a boundary the larger the amplitude on our seismic trace.
- Amplitude

Waves - Frequency

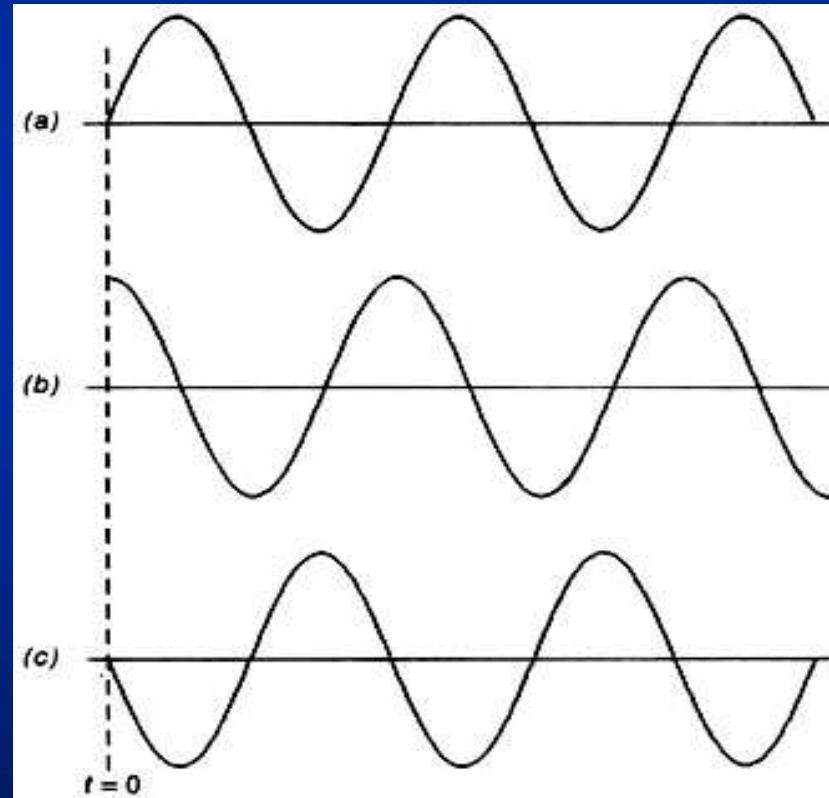
- Frequency is the repetition rate of a periodic waveform.
- The higher the frequency the more repetitious the waveform.
- Frequency



Waves - Phase

- Phase is the angle of lead or lag of a sine wave with respect to a reference.

- Phase



Sinusoid (sine wave) -
zero amplitude at $t=0$

Sinusoid (cosine wave)-
max amplitude at $t=0$
phase lead of 90° over
sine wave

Sinusoid - leads or lags
sine wave above by 180°

Signal Processing

- Waves
- Data sampling
- Seismic resolution
- Time and Frequency domain representation
- Convolution and Correlation

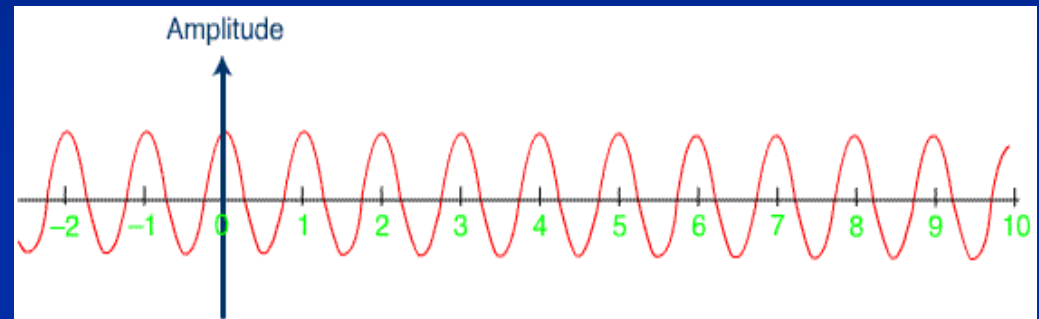
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- Explain the difference between a continuous and a discrete signal
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- Define the terms 'spatial sampling' and 'temporal sampling'
- Define the term 'Nyquist frequency' and describe how it can be calculated for a given sampling interval.
- Define the term 'aliasing' and describe how you would calculate an aliased frequencies new value for a given sample interval
- Describe the relationship between Nyquist frequency and aliasing
- Describe at least 3 ways in which temporal aliasing can decrease the quality of seismic data
- Define the term 'Nyquist wavenumber' and describe how it can be calculated for a given spatial sampling interval.

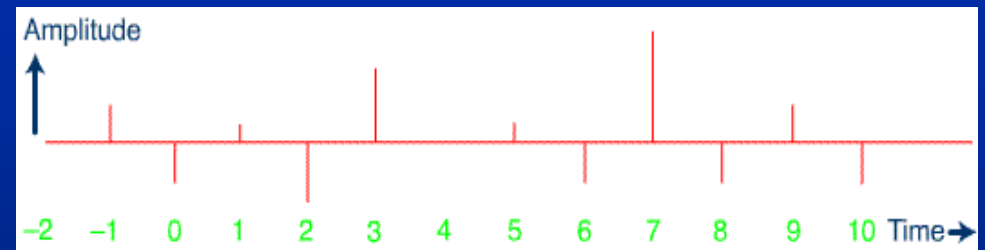
Data sampling

- A signal can be thought of as a quantity that carries a message and varies with time.

- There are 2 types of signal
 - continuous** has a signal value for all times



- discrete** has a signal value at certain times



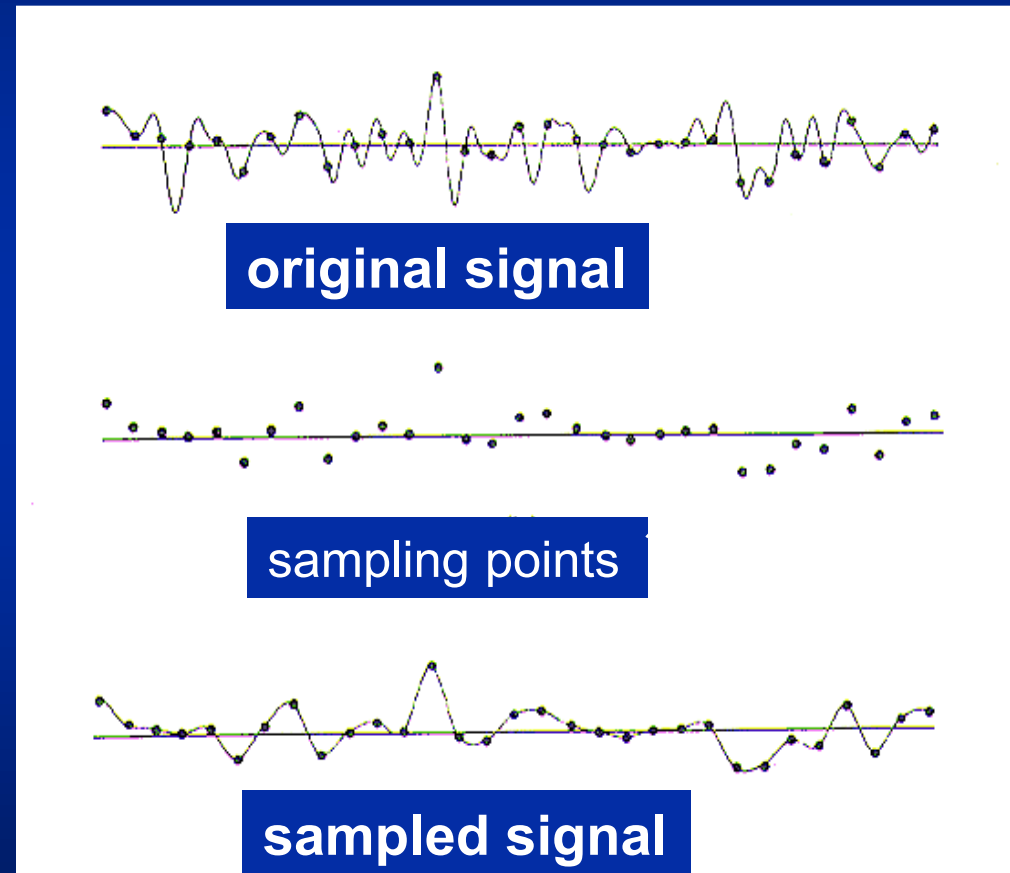
- A continuous signal becomes discrete when it is sampled

Data Sampling

- One of the first signal processing steps we perform is -
- Digital recording, i.e. the signal is not continuous but recorded at discrete intervals (time and space)
- Sample interval is the interval between readings, such as time between successive samples of a seismic trace or distance between traces of seismic data.
- More flexible than analog signal processing

Temporal Sampling

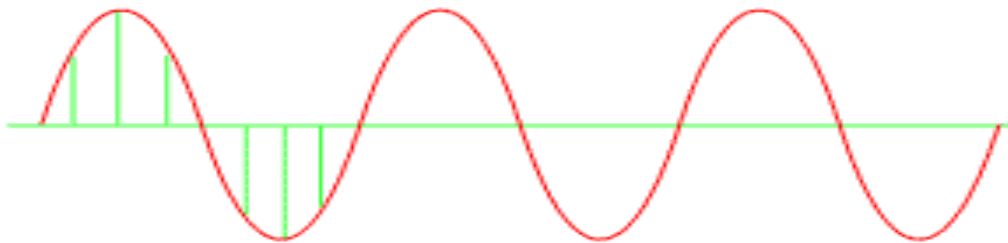
- To allow digital processing of the signal and to reduce the large data volumes recorded.
- Sampling reduces the high frequency content of the signal.
- Provided the frequencies lost are outside the useful signal frequency range then this is not a problem.
- The smaller the sampling interval the closer to the original will be the sampled signal.



Sampling Theorem

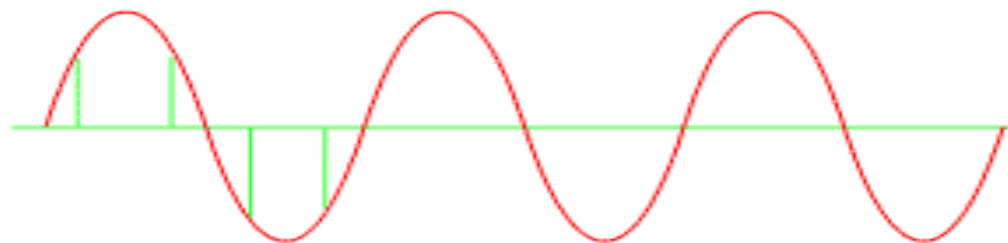
Consider, now, the frequency aspects of sampling.

Sampled 8 times per cycle



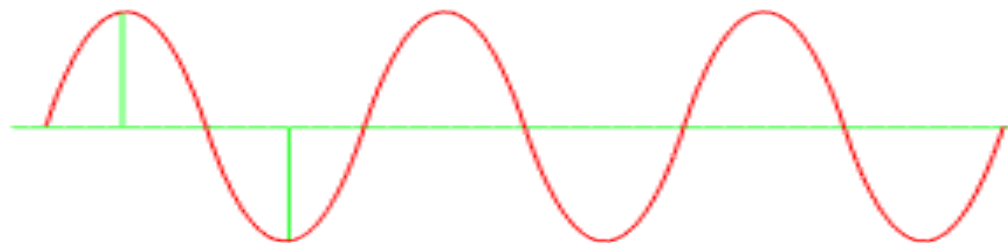
When we sample closely along a waveform, we should be able to construct a new wave from the samples, and the resultant wave will be the same as the original.

Sampled 4 times per cycle



In other words, our sampling scheme should preserve sufficient of the information from the input waveform as is necessary to reconstruct an identical wave, but should sample no more than is necessary to achieve this.

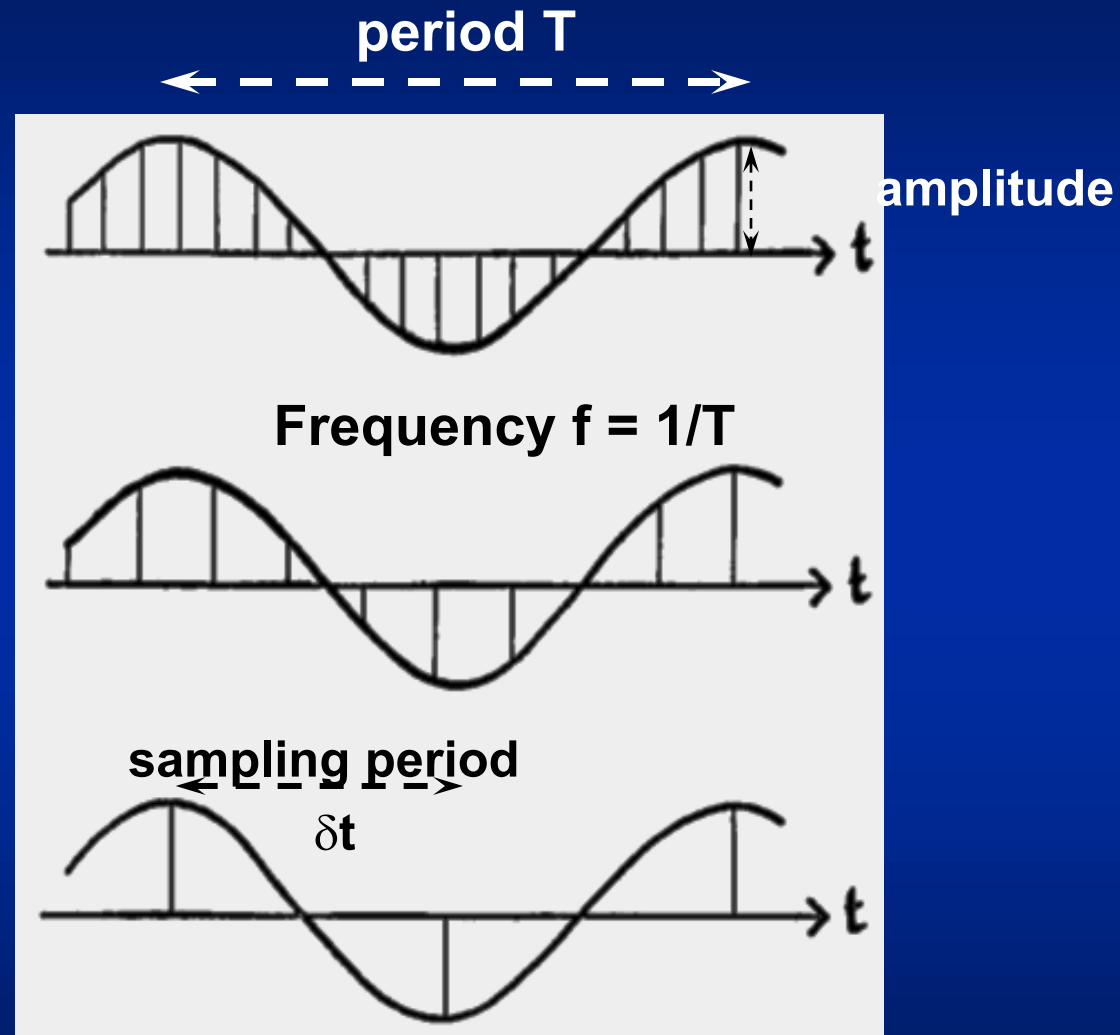
Sampled 2 times per cycle



Sampling theorem

- “Band-limited functions can be re-constructed from equi-spaced data if there are two or more points per cycle for the highest frequency present”
- This means that the sampling period ‘ δt ’ must be no more than half the signal period T .

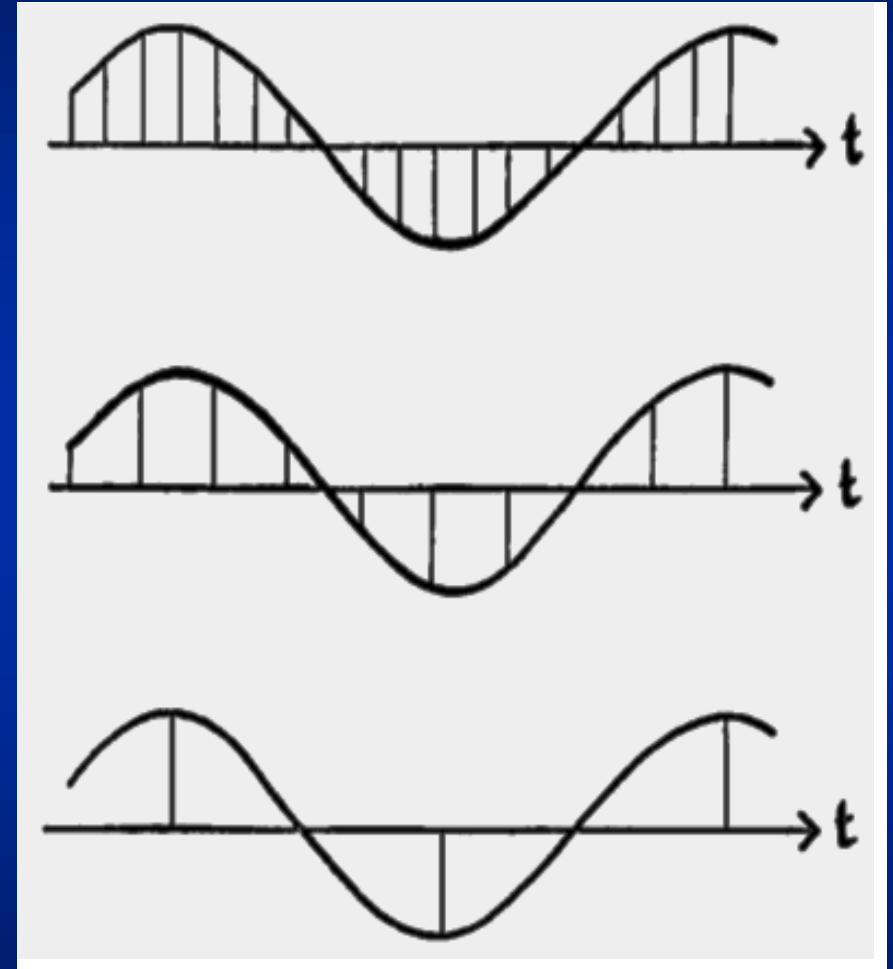
i.e. there must be :
at least two samples per cycle
for the highest frequency present



Nyquist Frequency

“The highest frequency that can accurately be reconstructed by using a particular sample interval”

$$f_{Nq} = 1/(2 \delta t)$$



Nyquist Frequency

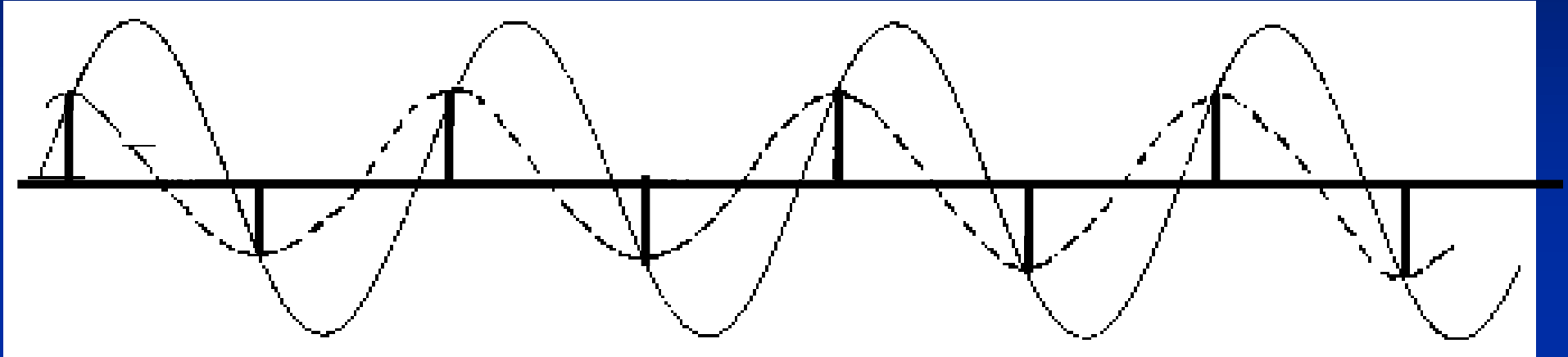
Suppose we are going to sample our signal every 4ms.

What is the highest frequency that we can accurately reconstruct

Using $f_{Nq} = 1/(2 \delta t)$

i. e. $1000/2 \times 4 = 125 \text{ Hz.}$

Sampling in practice

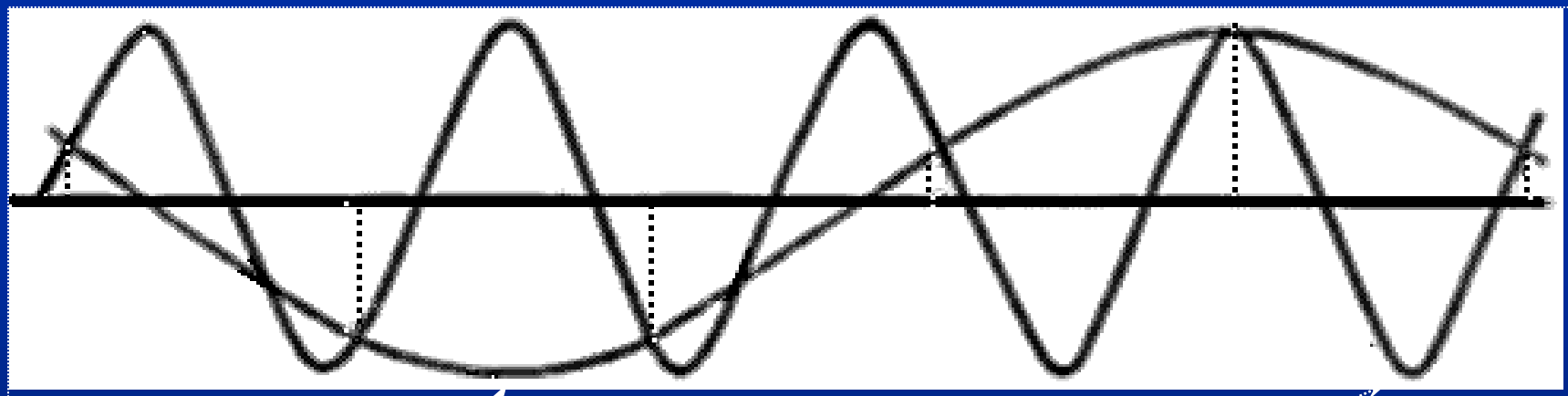


If a waveform is sampled right on its limits, i.e. twice per cycle, this can lead to a loss of amplitude information. Therefore the theoretical maximum for signal frequency based on a sampling interval is not achievable.

This means that in practice, for 4ms sampled data, the limit of usable signal frequencies is approximately up to 90 Hz.

Sampling - Aliasing

- Aliasing is a frequency ambiguity caused by under sampling of the continuous signal
- Occurs for frequencies above the Nyquist value and results in high frequencies appearing as low frequencies after the sampling



F_a aliased frequency

F_o original frequency

Sampling - Aliasing

So what is the new frequency of this aliased data

$$F_a = 2F_n - F_o$$

where F_a is the aliased frequency,
 F_n is the nyquist frequency
and F_o is the original frequency

What will the frequency be of the output signal if a 417 Hz input signal is sampled at 2 msec?

Sampling - Aliasing

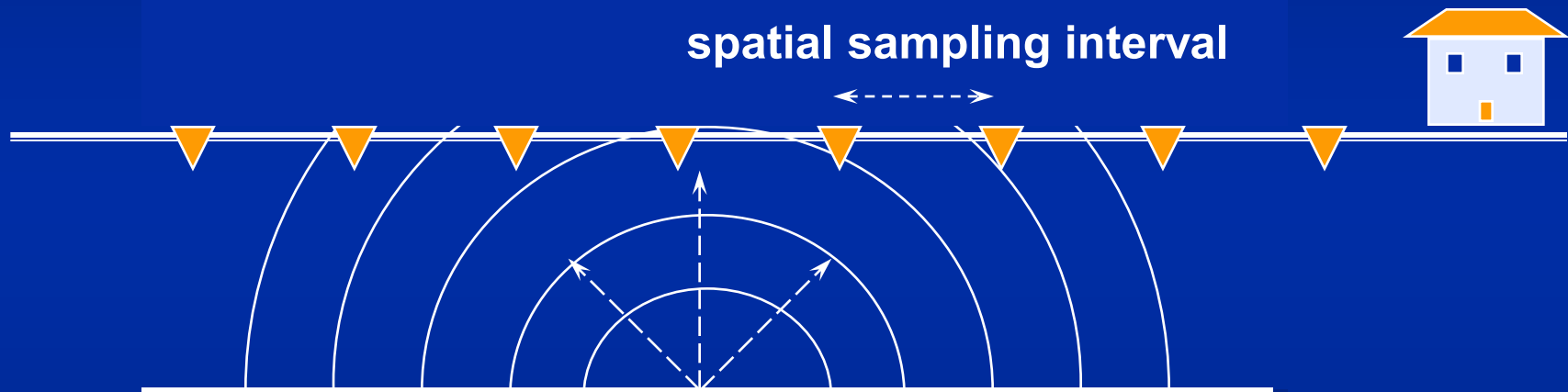
Temporal aliasing must be avoided otherwise:

- **Noise introduced into the data**
- **Amplitude distortion**
- **Loss of high frequency data**

Because of this, high frequencies above the maximum desired signal Frequency are removed by an anti-alias filter just before sampling. On the safe side, typically a 90 Hz high-cut filter is applied for 4 ms sampled data

Spatial Sampling

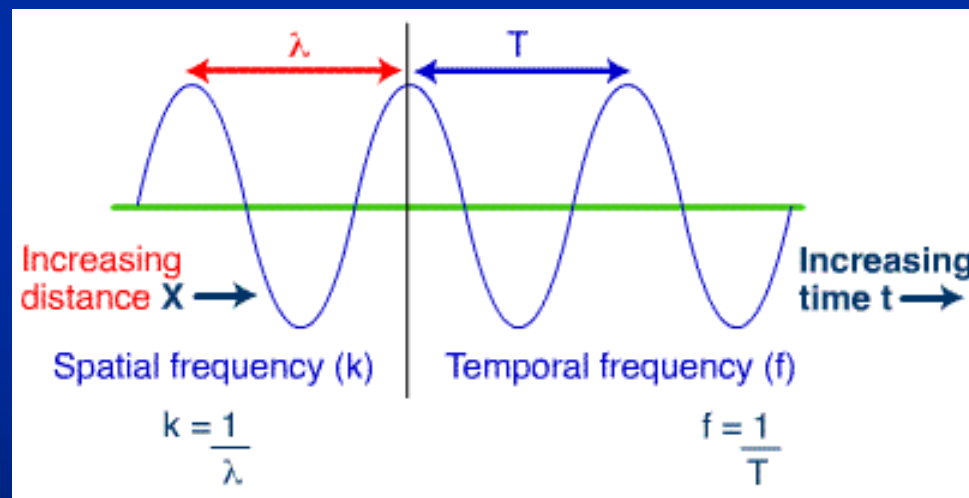
- The seismic signal is a continuous (analog) time function measured at detectors
- Although continuous in time, it is sampled in space (at the detector locations)



Reflected seismic signal

Spatial Sampling

Lets look at a waveform in space. We're now interested in the space required for a cycle of the wave ie wavelength



Spatial Sampling - Nyquist

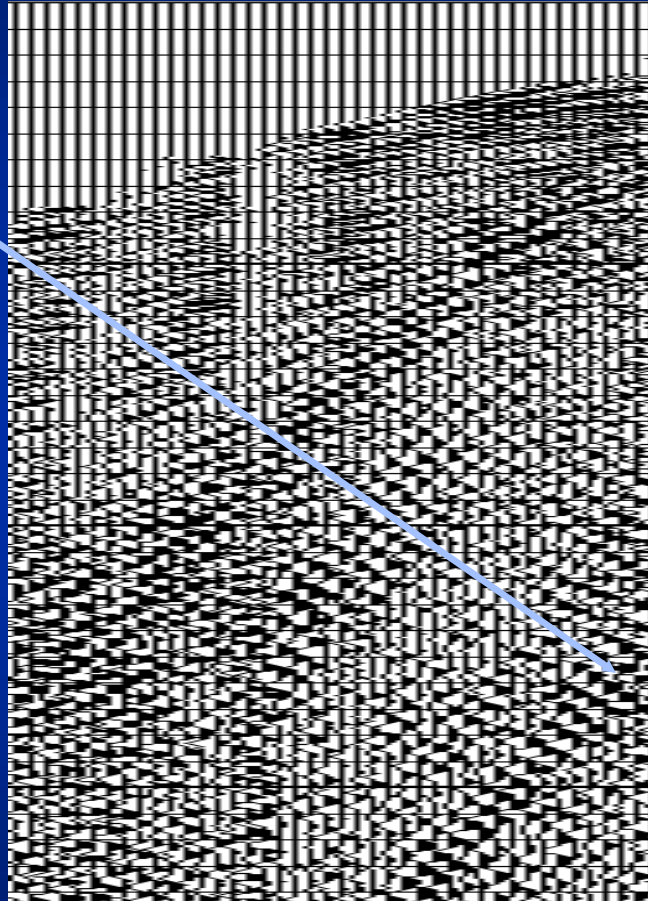
For spatial sampling we have a nyquist wavenumber, sampling above which can lead to aliased frequencies

$$k_{Nq} = 1/(2 \delta x)$$

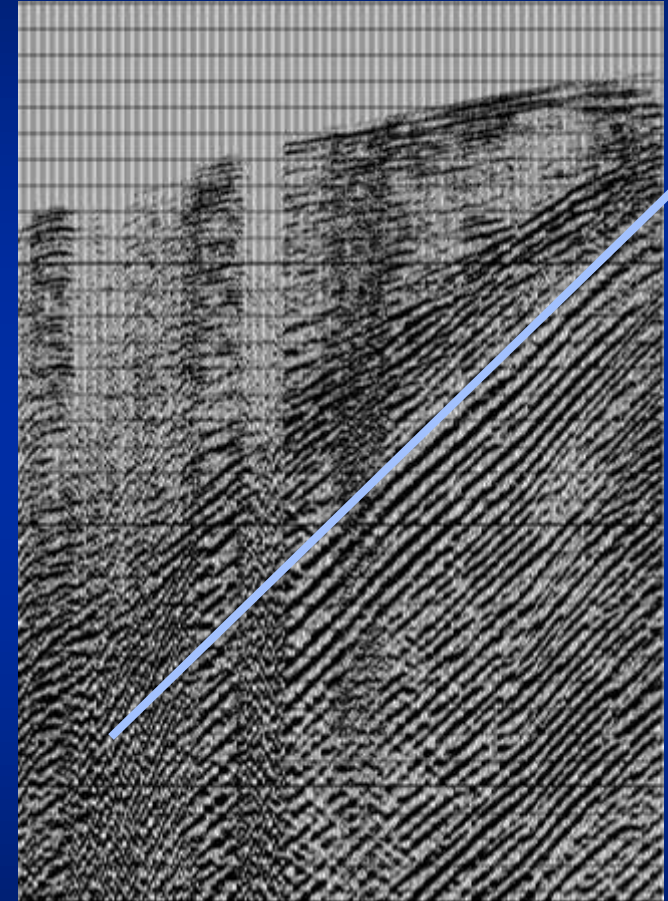
For receivers evenly spaced at 25m what is the k Nyquist?

Aliased Noise

On the left data is now aliased.
This means the data appears to be dipping in the opposite direction



Big distance between receivers



Small distance between receivers

Sampling - Aliasing

$$\text{temporal} = fNq = \frac{1}{2\Delta t}$$

Where 'f' is frequency in Hertz

$$\text{spatial} = kNq = \frac{1}{2\Delta x}$$

Where 'k' is wave-numbers in unit/distance

Signal Processing

- Waves
- Data sampling
- **Seismic resolution**
- Time and Frequency domain representation
- Convolution and Correlation

At the end of this Session, you will be able to:

- Define the term 'lateral (spatial) resolution'
- Define the term 'vertical (temporal) resolution'.
- Describe the Rayleigh criteria of resolution and show how it can be used to calculate vertical resolution.
- Define the term 'Fresnel zone' and calculate its radius for given velocity, frequency and travel time.

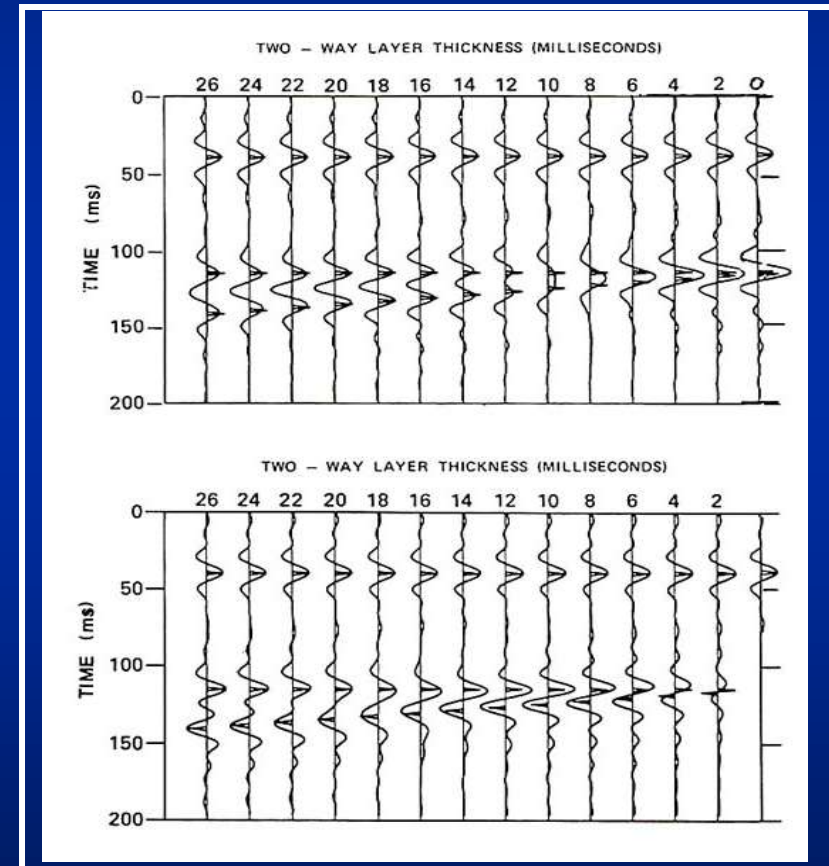
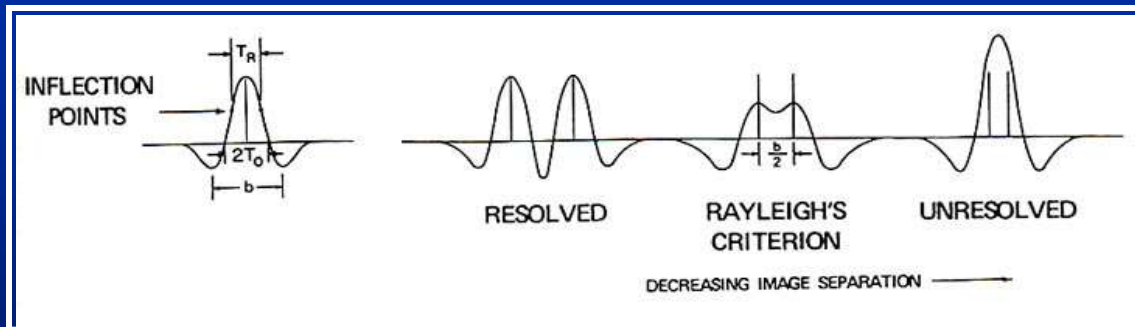
Vertical Resolution

Vertical resolution is the ability to distinguish between two separate reflectors that are very close together.

Vertical resolution is a function of the thickness of the subject layer, the frequency of the source wavelet as it propagates through the layer, and the velocity of the layer.

Rayleigh Criteria for Vertical Resolution

- Rayleigh criteria for resolution is defined as $\frac{\lambda}{4}$ and can be calculated as $V/4f$
- Below this separation, amplitude information must be used to separate events



Rayleigh Criteria for Vertical Resolution

Example:

Calculate typical wavelengths and resolution for the following:

Shale velocity = 3000 m/s

Dominant frequency on seismic at shale at T1ms, 70 Hz

$$\lambda = \frac{3000}{70} = 43\text{m} \Rightarrow \text{maximum resolution} \approx 11\text{m}$$

$$V = f\lambda$$

Limestone velocity = 5000 m/s

Dominant frequency on seismic at limestone at T2ms, 45 Hz

$$\lambda = \frac{5000}{45} = 111\text{m} \Rightarrow \text{maximum resolution} \approx 28\text{m}$$

$$\lambda = \frac{V}{f}$$

Rayleigh Criteria for Vertical Resolution

Example:

Calculate typical wavelengths and resolution for the following:

Shale velocity = 3000 m/s

Dominant frequency on seismic at shale at T1ms, 100 Hz

7.5m

$$V = f\lambda$$

Limestone velocity = 5000 m/s

Dominant frequency on seismic at limestone at T2ms, 100 Hz

12.5m

$$\lambda = \frac{V}{f}$$

Spatial Resolution

Spatial resolution is the ability to distinguish from where in the subsurface a reflection has originated.



Fresnel Zone

- The portion of a reflector from which reflected energy can reach a detector within one half wavelength of the first reflected energy

Spatial Resolution

Spatial resolution is improved by migration, because after migration we have a better idea of where our reflections came from.

$$R_{FR} = \frac{V}{2} \sqrt{\frac{t}{f}}$$

As time increases the Radius of fresnel zone increases

As frequency increases the Radius of fresnel zone decreases

As velocity increases the Radius of fresnel zone increases

Spatial Resolution

Example:

Calculate fresnel zone radius for the following

$$R_{FR} = \frac{V}{2} \sqrt{\frac{t}{f}}$$

Velocity	3000m/s	3000m/s	3000m/s	3000m/s
Time	1000ms	2000ms	3000ms	4000ms
Frequency	50Hz	50Hz	50Hz	50Hz
	212m	300m	367m	424m

Velocity	3000m/s	3000m/s	3000m/s	3000m/s
Time	1000ms	2000ms	3000ms	4000ms
Frequency	90Hz	90Hz	90Hz	90Hz
	158m	223m	273m	316m

Spatial Resolution

Example:

Calculate fresnel zone radius for the following

$$R_{FR} = \frac{V}{2} \sqrt{\frac{t}{f}}$$

Velocity	3000m/s	3000m/s	3000m/s	3000m/s
Time	1000ms	2000ms	3000ms	4000ms
Frequency	40Hz	40Hz	40Hz	40Hz
	237m	335m	410m	474m

Velocity	3000m/s	3000m/s	3000m/s	3000m/s
Time	1000ms	2000ms	3000ms	4000ms
Frequency	110Hz	110Hz	110Hz	110Hz
	143m	202m	247m	286m

Signal Processing

- Waves
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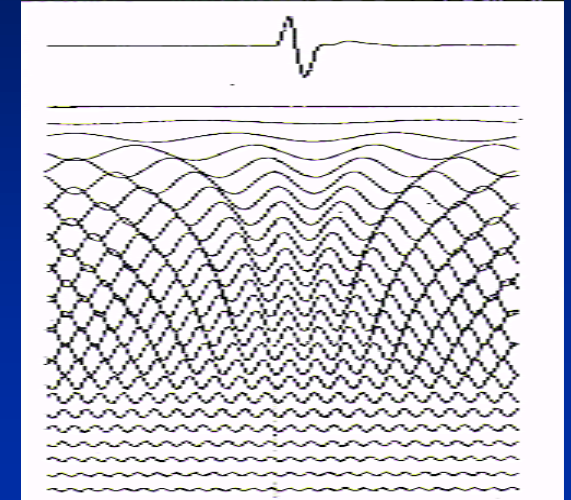
- Define the terms 'time domain' and 'frequency domain'.
- Define the term 'Fourier series'.
- Define the terms 'fundamental frequency' and 'harmonic frequency'.
- Define the terms 'amplitude spectrum' and 'phase spectrum' and describe their components.
- Describe and compare the definitions of each of the following transforms: Fourier Transform, Discrete Fourier Transform and Fast Fourier Transform.
- Match time domain representations with their frequency domain equivalents

Time and Frequency domain representation

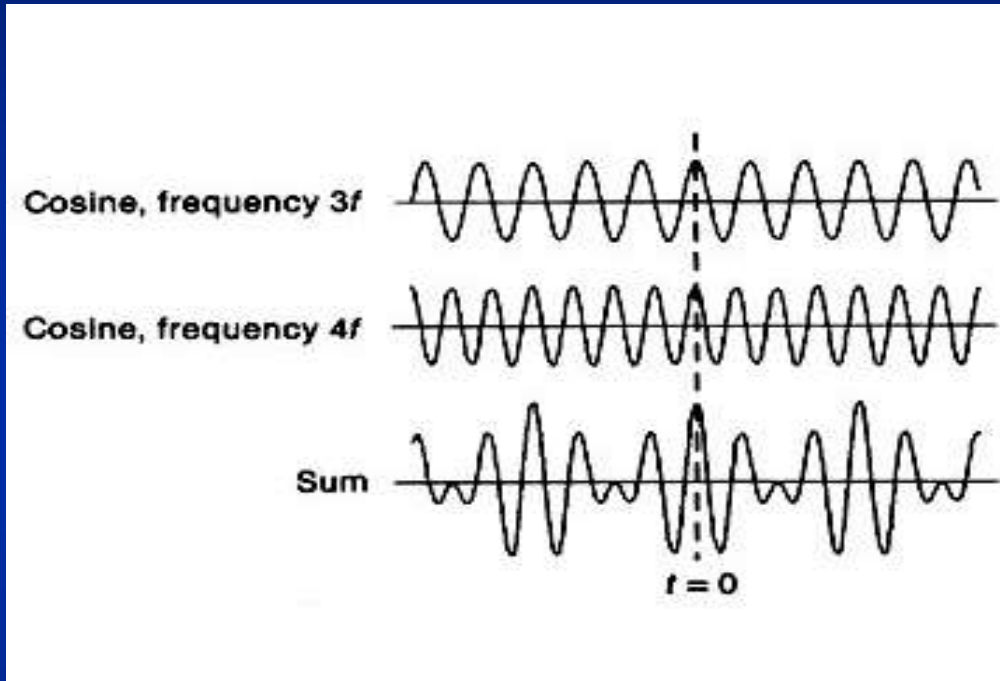
- The bridge between the time domain and the frequency domain is defined by Fourier
- The Fourier Transform is simply a mathematical process that allows us to take a function of time (a seismic trace) and express it as a function of frequency (amplitude and phase spectra).
- Any repetitive waveform can be represented in the frequency domain by a pair of spectra
- The pair consists of an amplitude and a phase spectra

Fourier Series

- Any *repetitive* waveform, however complicated, may be viewed as the addition of Sine (or Cosine) waves whose frequencies are integral multiples of that of the basic repetition
- The basic repetition is called the 'fundamental' and is always the highest common factor of any composite waveform
- The frequencies which are 2, 3, 4 times that of the fundamental are called *harmonics*



Fourier Series



Third harmonic

Fourth harmonic

Basic repetition frequency
is f , the fundamental frequency

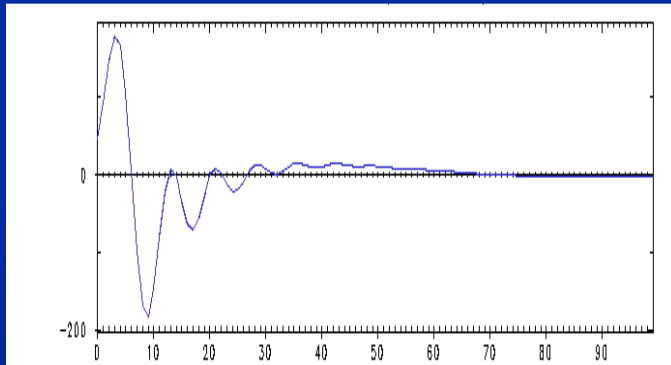
The fundamental frequency makes itself evident in the composite waveform (bottom) even though it is not present among the third and fourth harmonic components shown above.

The Fourier Transform

Time domain

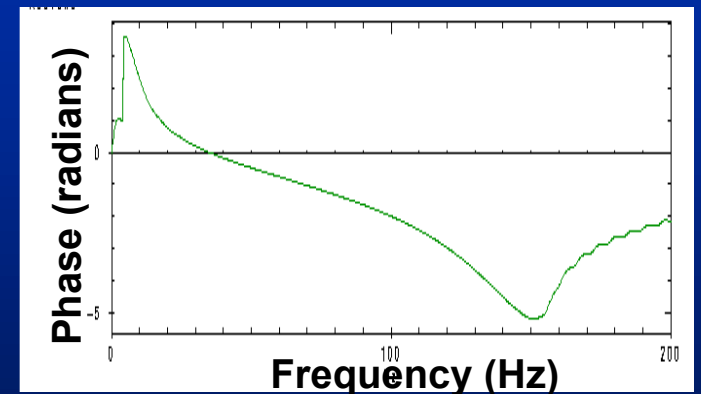
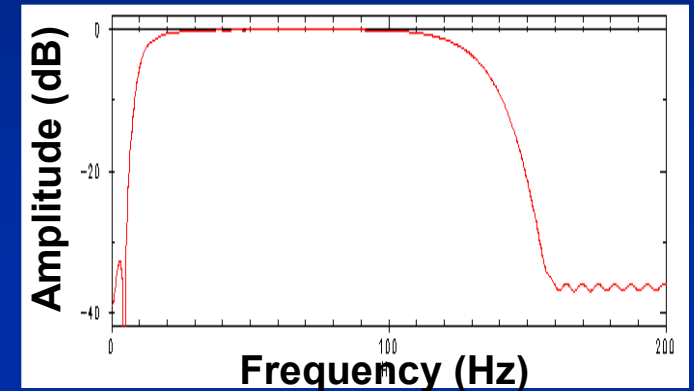


Frequency Domain



Time (ms)

1D fourier
transform



Continuous Fourier Transform

Continuous time function $X(t)$ converted into its frequency domain representation $F(f)$

$$X(f) = \int_{-\infty}^{\infty} X(t) \exp(-i2\pi ft) dt$$

and the inverse transform

$$X(t) = \int_{-\infty}^{\infty} X(f) \exp(i2\pi ft) df$$

Discrete Fourier Transform

For a discrete (i.e. sampled) function $x_k = x_0 + x_1 + x_2 \dots x_{N-1}$
Discrete Fourier Transform (DFT) –

$$\mathbf{X(f)} = \sum_k X_k \exp - (i2\pi f k \Delta t)$$

where $k = 0, 1, 2, \dots, N-1$ (N =number of samples) and the trace is one period of a periodic signal

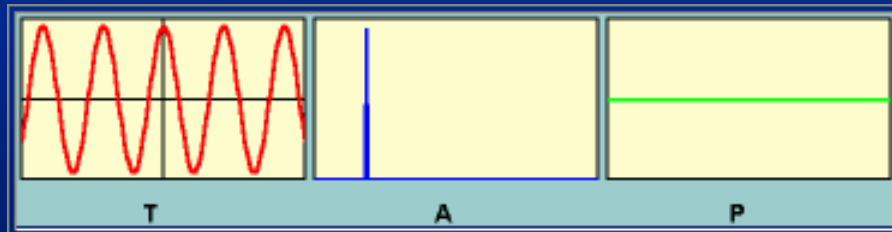
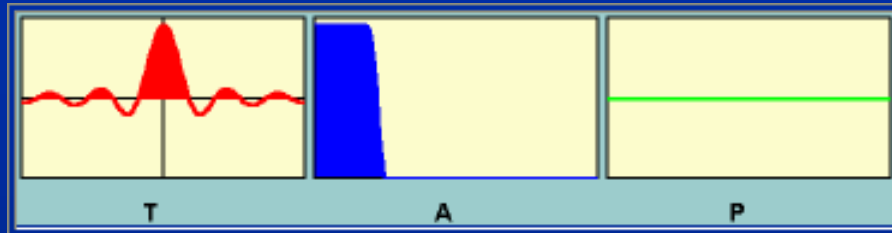
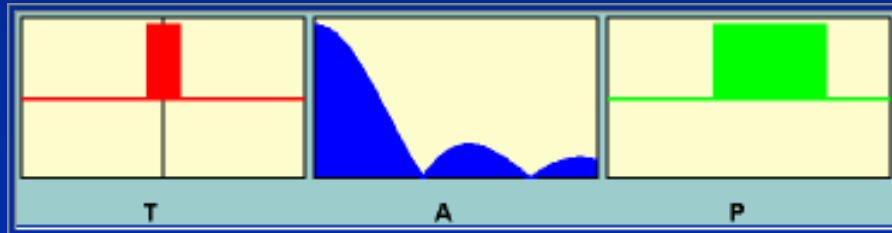
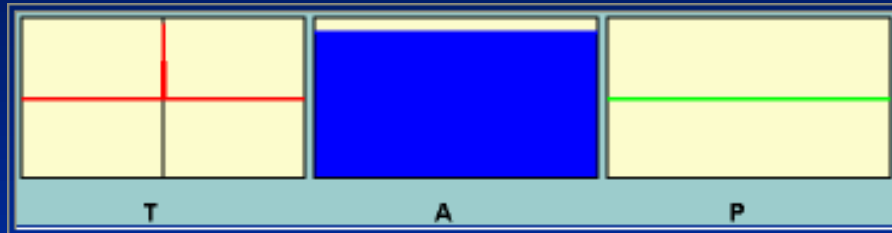
Fast Fourier Transform

The **Fast Fourier Transform (FFT)** is the standard implementation of the DFT on computers.

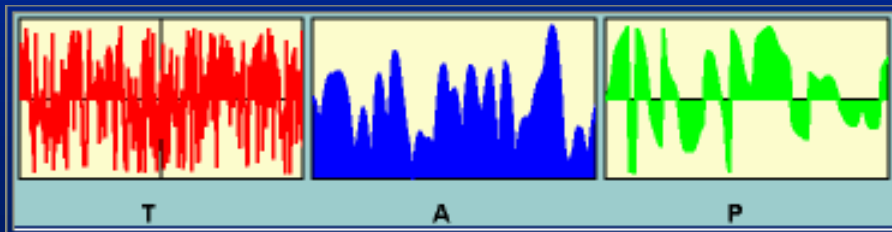
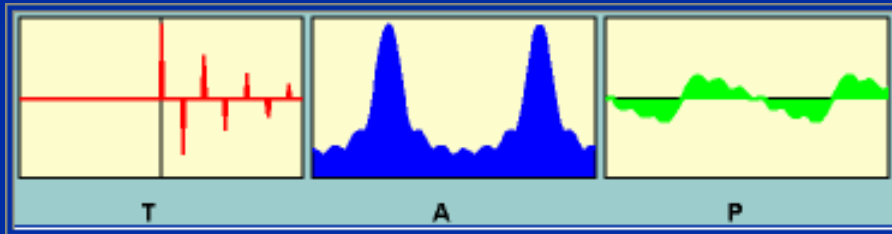
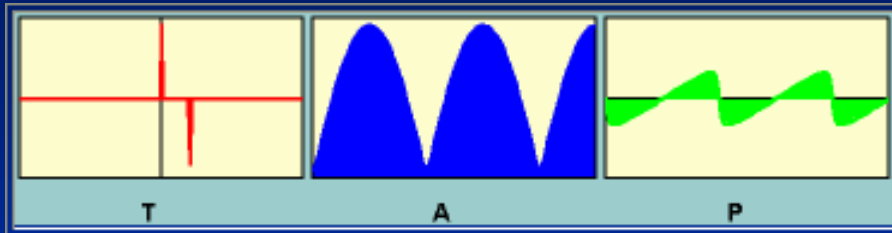
The number of calculations are reduced.

For FFT number of samples, N , must be a power of 2
e.g. 2 4 8 16 32 64 128.....

Example Fourier Transforms



Example Fourier Transforms



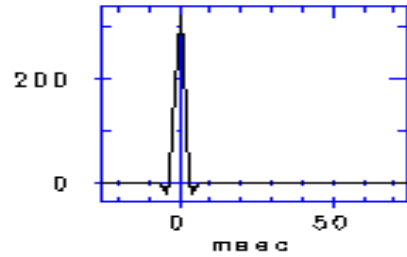
Time Domain



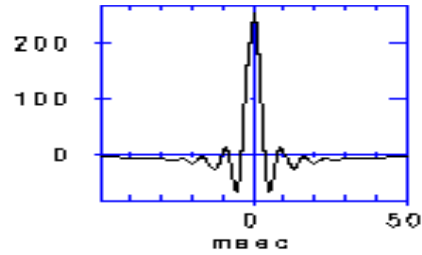
Frequency Domain

Time (ms) versus Amplitude (mvolts)

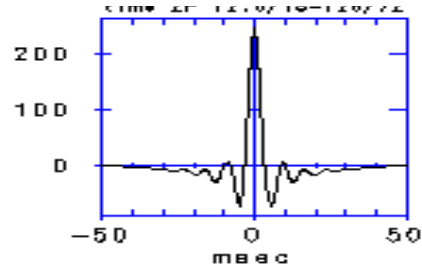
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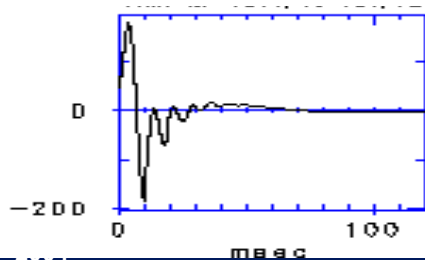
ZP 8/18 - 128/72



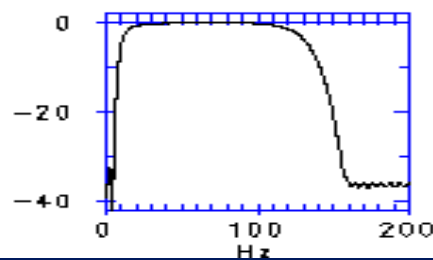
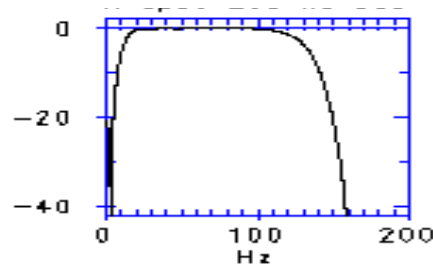
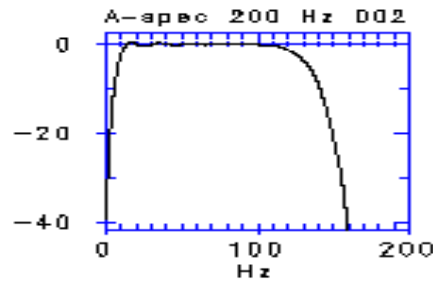
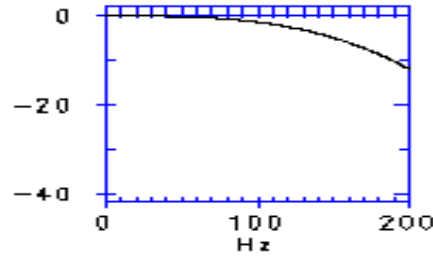
ZP 12/18 - 128/72



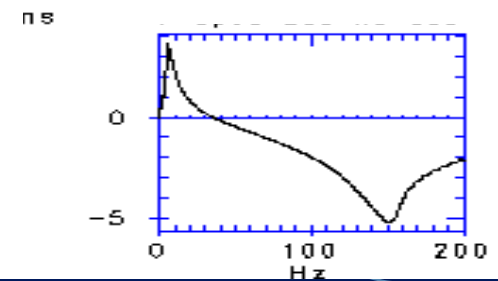
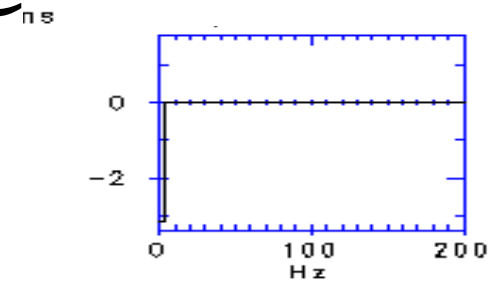
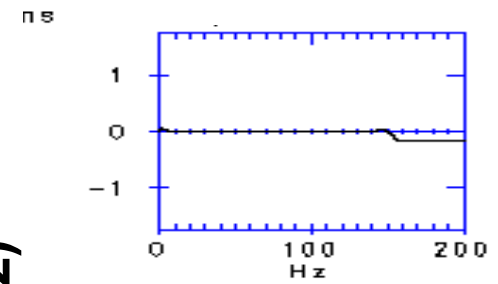
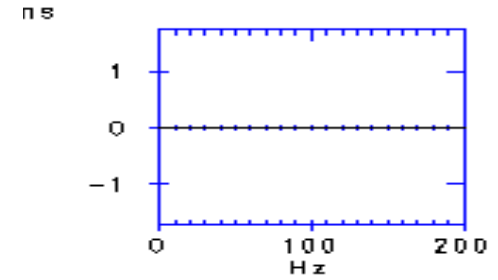
MP 12/18 - 128/72



Amplitude (dB) versus Frequency (Hz)



Phase (radians) versus Frequency (Hz)



Signal Processing

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- Convolution and Correlation

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- Describe how to correlate two wavelets
- Describe the equivalent of correlation in the frequency domain
- Describe how cross correlation and auto-correlation can be used in data processing
- Define the term 'convolution'
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Correlation

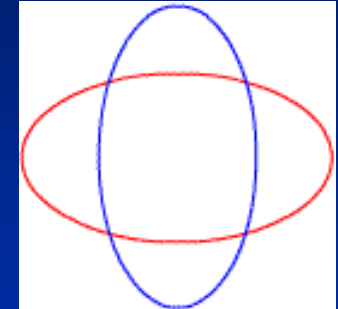
By correlation, we really mean the comparison of two, individual things so as to ascertain the degree of similarity that exists between them

An application of correlation outside signal theory is the comparison of fingerprints

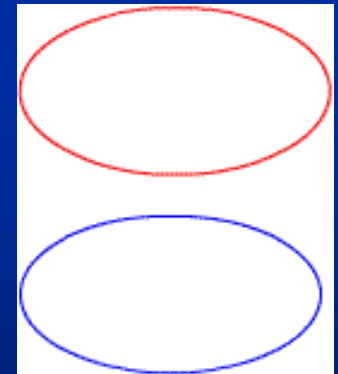


Correlation

Can you guess the degree of correlation of the following 2 ellipses

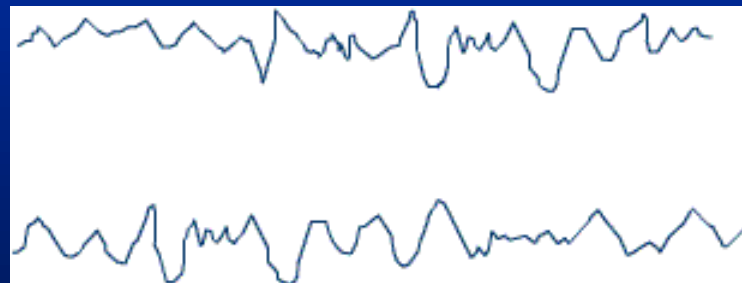


Lets rotate them. Now are they exactly the same?



Correlation

- Provides a means of measuring the similarity between two waveforms.
- Correlation is used to compare two signals to see how similar they are to each other for differing, relative positions. Essentially, one signal slides past the other signal and their similarity is measured.
- When we correlate two waveforms that are different we call it 'Cross-correlation'
- When a waveform is cross-correlated with itself, we call it 'Auto-correlation'



Correlation

$$f_t = \sum_{\tau=-\infty}^{\tau=+\infty} x_t \cdot h_{t+\tau}$$

Which, roughly translated, means:-

- 1.Lay the filter next to the seismic trace.
- 2.Cross-multiply each sample in the filter by each sample in the trace, and add the results to give one output sample.
- 3.Move the filter along one sample and do it again.

This is **CORRELATION!**

Correlation

1) Multiply sample n (B) with sample 1 (A) and output value

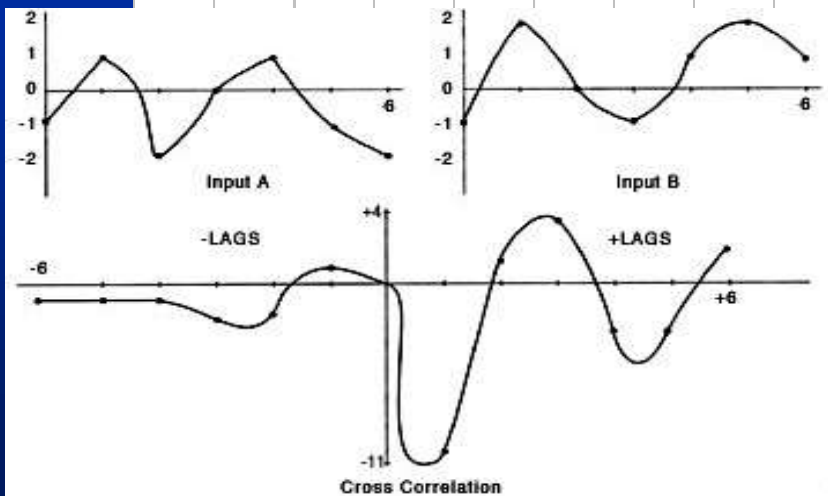
2) Slide (B) along 1 sample then multiply sample n (B) with sample 2 (A) and add to the product of sample n-1 (B) and sample 1 (A) then output

3) Continue to slide (B) across 1 sample and cross multiply with samples from (A), adding their products then output.

	A	-1	0	1				
1	-1	-1	B					1
	1	-1	-1					1
		1	-1	-1				-2
			1	-1	-1			-1
				1	-1	-1		1

Correlation

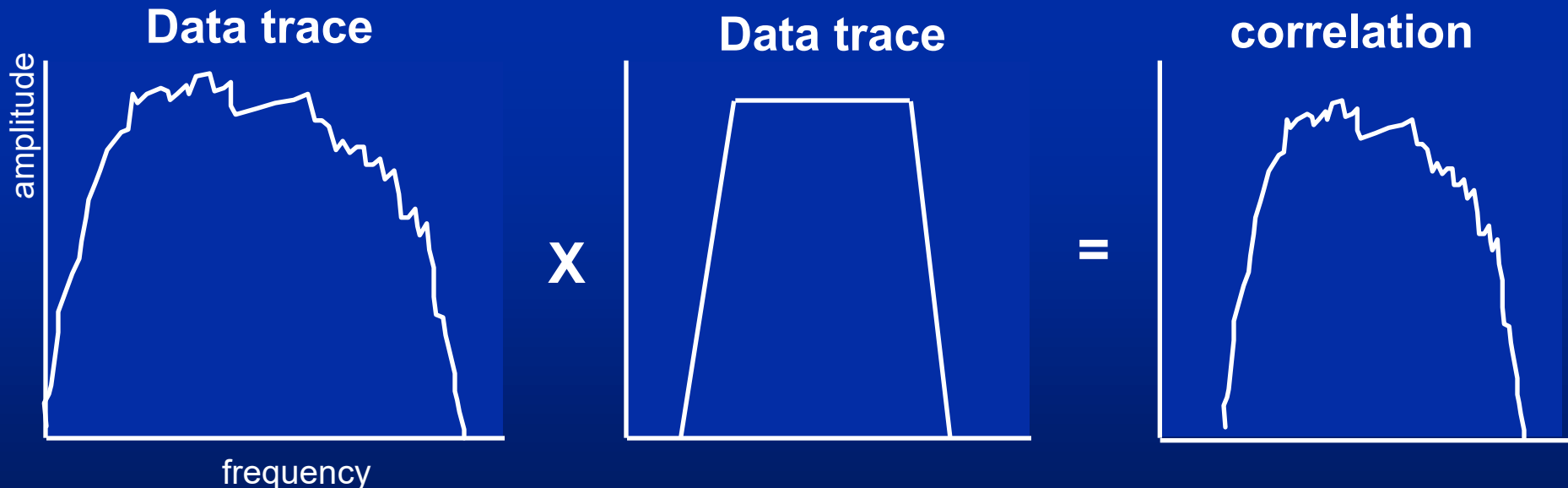
					A	-1	1	-2	0	1	-1	-2								
-1	2	0	-1	1	2	1	B													-1
	-1	2	0	-1	1	2	1													-1
		-1	2	0	-1	1	2	1												-1
			-1	2	0	-1	1	2	1											-2
				-1	2	0	-1	1	2	1										-2
					-1	2	0	-1	1	2	1									1
						-1	2	0	-1	1	2	1								0
							-1	2	0	-1	1	2	1							-11
								-1	2	0	-1	1	2	1						1
									-1	2	0	-1	1	2	1					4
										-1	2	0	-1	1	2	1				-3
											-1	2	0	-1	1	2	1			-3
												-1	2	0	-1	1	2	1		2



Correlation

(in the frequency domain)

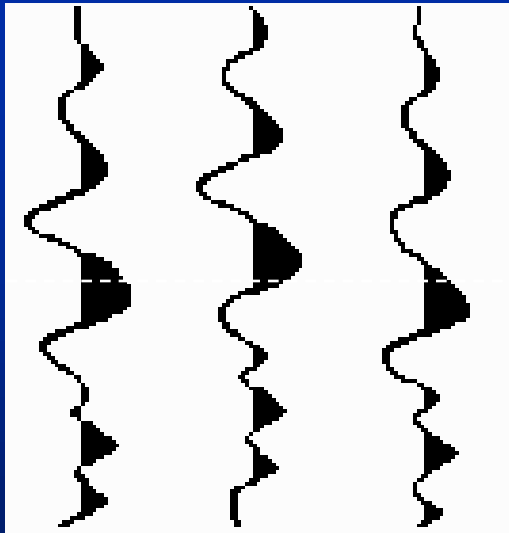
Correlation in the time domain $\equiv$ multiplying amplitude spectra and subtracting phase spectra in the frequency domain



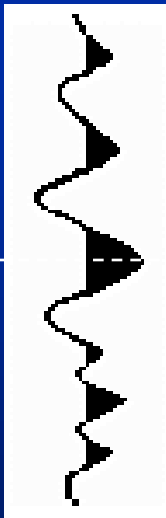
Correlation

- Used in static derivation programs to compute timing shifts between traces on land data (time shifts caused by near surface geology variations).

uncorrected traces



pilot trace

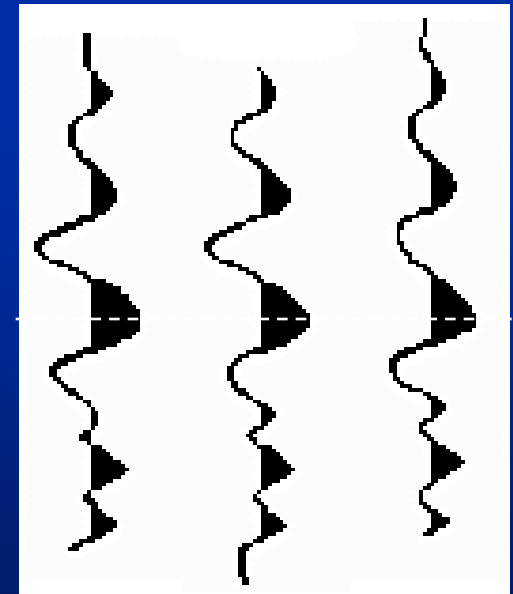


cross
correlate



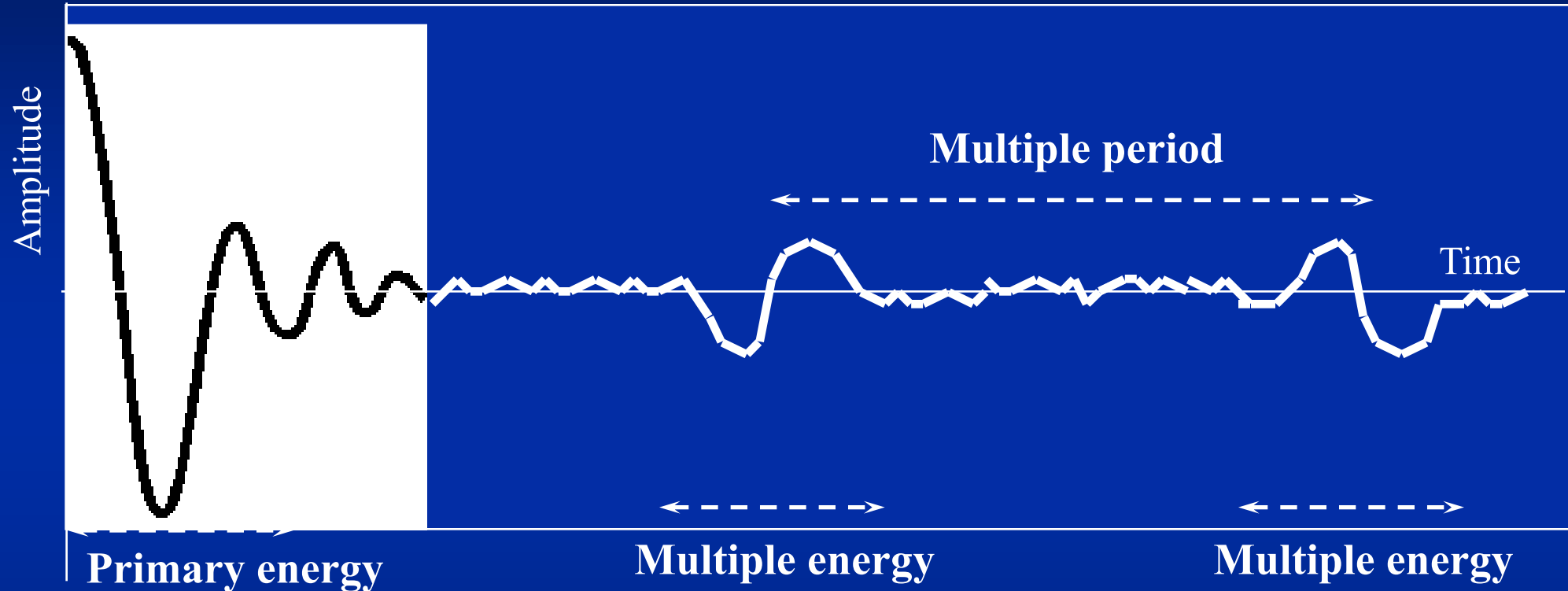
compute time shifts
and correct traces

corrected traces



Autocorrelations & Multiples

Autocorrelation is the cross correlation of a trace with itself



The autocorrelation function of a seismic signal is an effective tool for determining the presence and time lag (duration) of reverberations/multiples.

Convolution

Convolution is the change of a wave shape as a result of passing it through a linear filter

When a signal passes through any filter (such as the earth), it is replicated many times with different amplitudes and time delays, by the filter. Assuming that the signal, itself, does not alter with the passing of time (ie. it is time shift invariant), then the filter produces a linear superposition of these copies of the signal. The mathematical description of this process is known as convolution.

Convolution

$$f * g \equiv \int_0^t f(\tau)g(t - \tau) d\tau,$$

Which, roughly translated, means:-

1. Time-reverse the filter (the $-T$ bit).
2. Lay it next to the seismic trace.
3. Cross-multiply each sample in the filter by each sample in the trace, and add the results to give one output sample.
4. Move the filter along one sample and do it again.

This is **Convolution!**

Convolution

Consider the following wavelet



Let's take a look
at the effect of
convolution
with a filter
(3, -1, 2)
where the 3 is
a delay of 0 time
units, the -1
is a delay of 1
time unit and
the 2 is a delay
Of 2 time units.

Filter

3

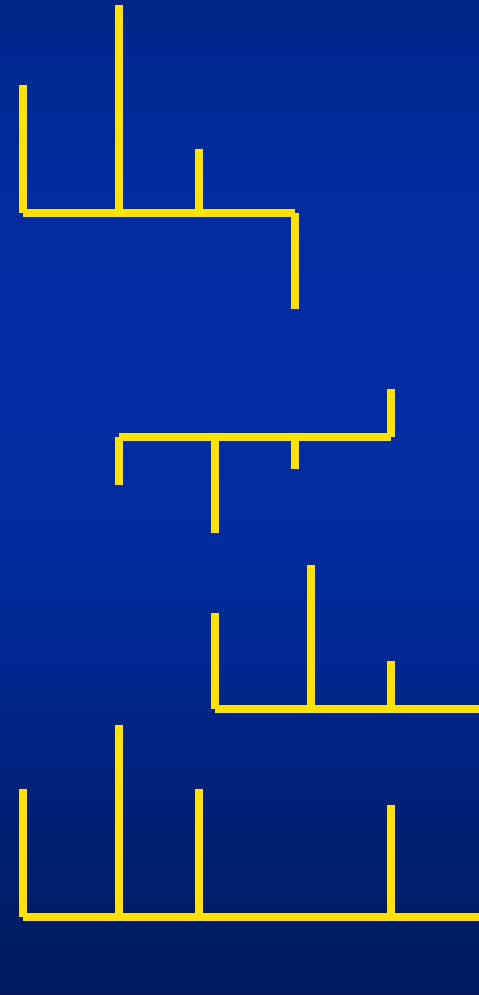
-1

2

+

+

=



Convolution

1) Time Reverse B

2) Multiply sample n (B) with sample 1 (A) and output value

3) Slide (B) along 1 sample then multiply sample n (B) with sample 2 (A) and add to the product of sample n-1 (B) and sample 1 (A) then output

4) Continue to slide (B) across 1 sample and cross multiply with samples from (A), adding their products then output.

	A	-1	0	1				
-1	-1	1	B					-1
	-1	-1	1					1
		-1	-1	1				2
			-1	-1	1			-1
				-1	-1	1		-1

Convolution

					A	-1	1	-2	0	1	-1	-2								
1	2	1	-1	0	2	-1	B													1
	1	2	1	-1	0	2	-1													-3
		1	2	1	-1	0	2	-1												4
			1	2	1	-1	0	2	-1											-3
				1	2	1	-1	0	2	-1										-3
					1	2	1	-1	0	2	-1									4
						1	2	1	-1	0	2	-1								-1
							1	2	1	-1	0	2	-1							-8
								1	2	1	-1	0	2	-1						0
									1	2	1	-1	0	2	-1					3
										1	2	1	-1	0	2	-1				-3
											1	2	1	-1	0	2	-1			-5
												1	2	1	-1	0	2	-1		-2

Convolution

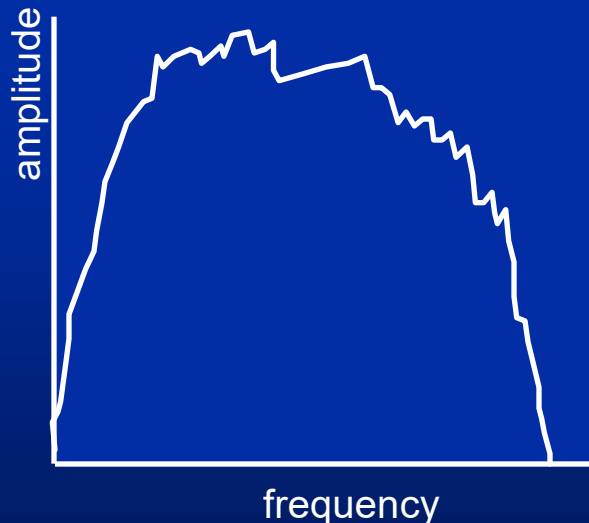
(in frequency domain)

Filtering in the time domain
by convolution

≡

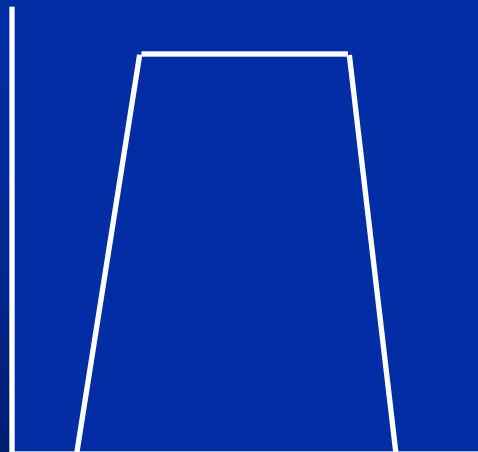
Filtering in the frequency domain
by multiplying amplitude spectra
and adding phase spectra

INPUT



FILTER

X



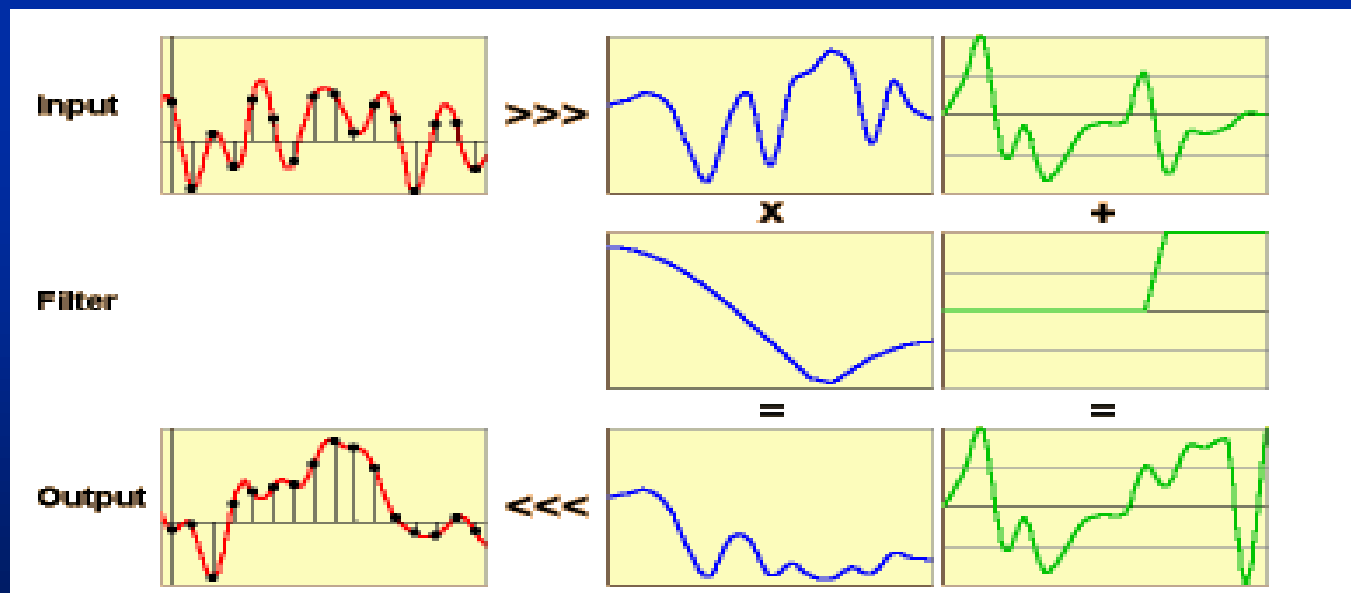
=

OUTPUT

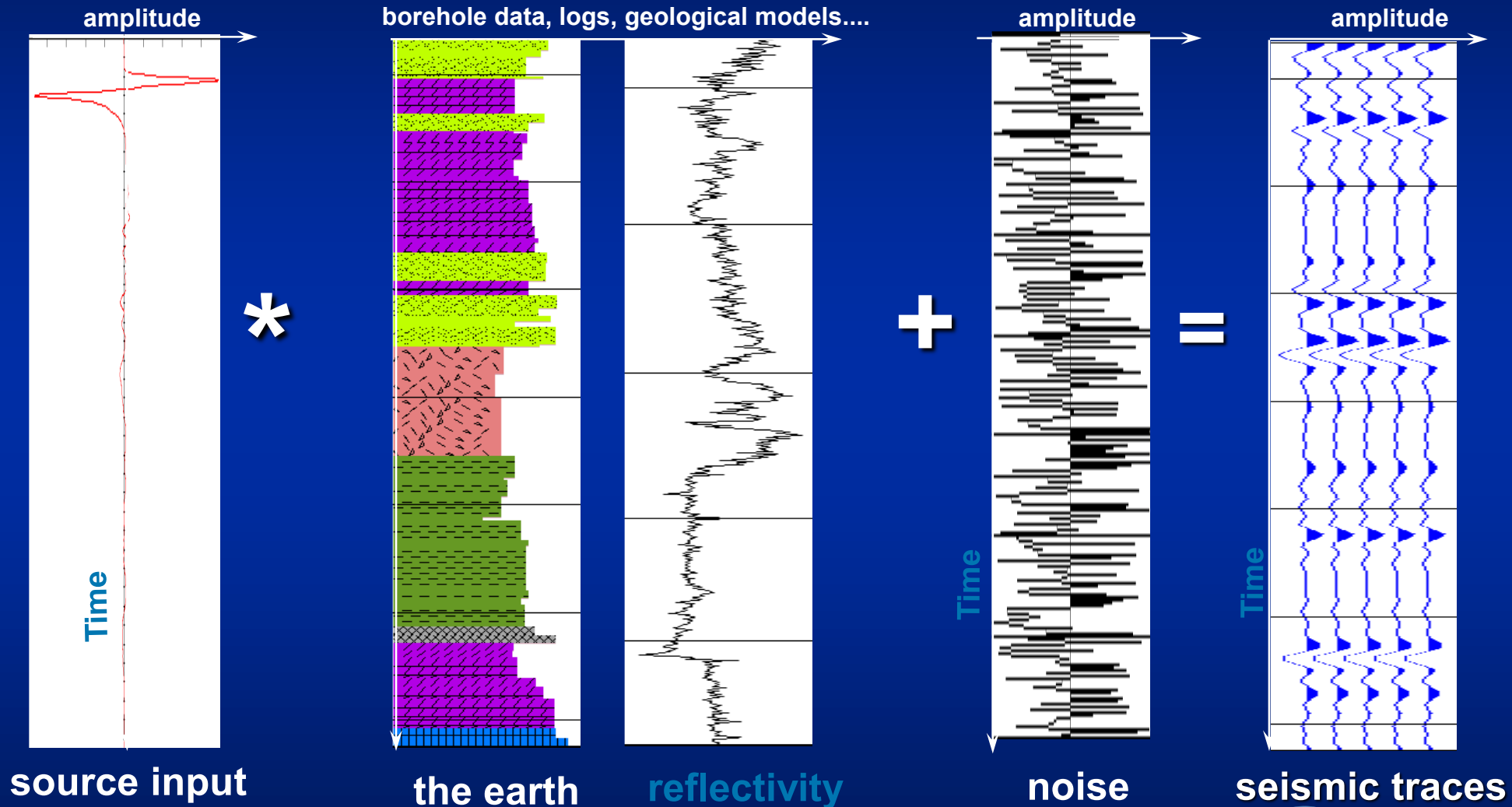


Digital Filtering

- A filter is essentially a relationship between input and output for some system.
- If a signal s_t is filtered by a filter, with impulse response f_t then the output is the convolution of f_t and s_t .



Convolutional Model



The Convolutional Model

- The source signal links with the reflection sequence to produce a trace which is a superposition of wavelets scaled and delayed by the reflection sequence.
- Superposition of this type is another name for **convolution**.
- Hence the name - **Convolutional Model**.

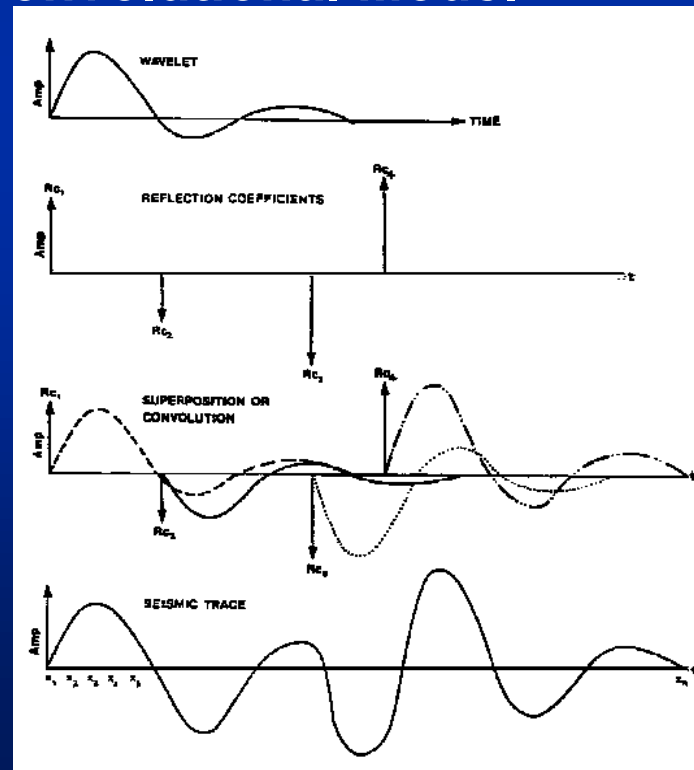
$w(t)$

$*$

$r(t)$



$s(t)$



Input wavelet

Earth's reflectivity function

Superposition/Convolution

Output trace

